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RE	Restricted to a group specified by the Consortium (including the Commission Services)	
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If this report is not to be made public, please state here why: not applicable.

Contribution to project objectives – with this deliverable, the project has contributed to the achievement of the following objectives (from Annex I / DOW, Section B1.1.):

N.º	Objective	Yes	No
1	Reduce uncertainties in our knowledge of the functioning of Tropical Atlantic (TA) climate, particularly climate-related ocean processes (including stratification) and dynamics, coupled ocean, atmosphere, and land interactions; and internal and externally forced climate variability.	X	
2	Better understand the impact of model systematic error and its reduction on seasonal-to-decadal climate predictions and on climate change projections.	X	
3	Improve the simulation and prediction TA climate on seasonal and longer time scales, and contribute to better quantification of climate change impacts in the region.		
4	Improve understanding of the cumulative effects of the multiple stressors of climate variability, greenhouse-gas induced climate change (including warming and deoxygenation), and fisheries on marine ecosystems, functional diversity, and ecosystem services (e.g., fisheries) in the TA.		
5	Assess the socio-economic vulnerabilities and evaluate the resilience of the welfare of West African fishing communities to climate-driven ecosystem shifts and global markets.		

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Deviation from planned efforts for this deliverable: none to our best knowledge.

Executive Summary:

WP10 (Statistical methods to assess and improve forecast of Tropical Atlantic variability) aims at (1) developing a Bayesian hierarchical modeling strategy to re-calibrate forecasts and improve prediction of Tropical Atlantic variability (TAV) and its impact, (2) developing a statistical scheme to predict sea-surface temperature (SST) anomalies in remote regions associated with TAV, and (3) assessing the ability of state-of-the-art climate models (CMIP5) to reproduce climate variations over the Tropical Atlantic Sector (including surrounding continents) over the 20th century. To the first purpose, Bayesian hierarchical modeling strategies are being developed and used at the Ca' Foscari University of Venice to improve TAV predictions and re-calibrate simulated data for bringing them into line with measurements. The main theoretical advantage of a Bayesian strategy is that it encapsulates the uncertainties involved in the estimation of all model parameters and these uncertainties are properly taken into account in the predictions and re-calibrations. This deliverable consists of a report about the statistical Bayesian models produced within WP10 and practical information about the associated software.

Specifically, three main statistical models have been developed within a Bayesian framework, which target different tasks of WP10, including development of a general methodology to obtain joint

projections of climate indexes, based on ensembles of global climate model output and historical observations and to re-calibrate simulated data to bring them into line with the measurements (Task 10.1); optimal merging of output distributions of a multi-model simulation ensemble to produce deeper statistical confidence on biases (Task 10.2), optimize predictions of climate processes underlying observed physical and/or biogeochemical parameters (Task 10.3). The first branch is focused on estimation of the *temporal* component of systematic model errors through structural decomposition, using the evolution of SST drifts in the Tropical Atlantic region from decadal climate predictions as a test bed and case study; the second branch is focused on the *spatial* assessment of the bias in a multi-model ensemble, using near-surface air temperatures over the Tropical Atlantic region from CMIP5 historical simulations as test bed; the third branch is focused on the *spatio-temporal* assessment of the bias in a multi-model ensemble, which is illustrated using the same case study as the second branch. This deliverable further includes relevant R and matlab codes, when these have not yet been made publicly available.

The present document consists of three main sections:
1. A Bayesian approach to estimation of climate model drift and bias
2. Bayesian hierarchical spatial assessment of the bias in a multi-model ensemble
3. Spatio-temporal assessment of climate model bias: a Bayesian approach

Section 1 contains material from the publication: Zanchettin D., Gaetan C., Arisido M. W., Modali K., T. Toniazzo, N. Keenlyside, Rubino A. (2017) Structural decomposition of decadal climate prediction errors: A Bayesian approach. Scientific Reports 7, 12862, doi:10.1038/s41598-017-13144-2.
Section 2 contains material from the publication: Arisido, M. W., C. Gaetan, D. Zanchettin, A. Rubino (2017) A Bayesian hierarchical approach for spatial analysis of climate model bias in multi-model ensembles. Stoch. Environ. Res. Risk. Assess., in press, doi:10.1007/s00477-017-1383-2
Section 3 contains material from the manuscript "Spatio-temporal quantification of climate model errors in a Bayesian framework" by Maeregu Woldeyes Arisido, Carlo Gaetan, Davide Zanchettin, Jorge Lopez Parages and Angelo Rubino, under review in Environmental and Ecological Statistics (submission ID: EEST-D-17-00076)

1. A Bayesian approach to estimation of climate model drift and bias

This section describes the statistical model used in Zanchettin et al. (2017). The reader is referred to Zanchettin et al. (2017) and to PREFACE-MS34 for additional details about the rationale of the approach and for illustrative examples.

The climatology simulated by coupled climate models used in contemporary decadal climate prediction systems is affected by systematic biases compared to observations with respect to mean state, seasonal cycle and interannual internal variability. Decadal climate forecasts with full-field initialized coupled climate models are therefore affected by a systematic growing error signal – so-called drift - that develops due to the adjustment of the simulations from the assimilated state consistent with observations to a state consistent with the biased model's climatology. Model drifts thus reflect a fundamental source of uncertainty in decadal climate predictions.

We have thus developed a state-space model within a Bayesian hierarchical framework for a process-oriented statistical assessment of systematic decadal climate prediction errors. Specifically, the state-space model is used to attribute temporal changes observed in the hindcast errors (data) to the "discriminant effects" underlying these changes (process), which include systematic components such as drift and climatological biases in mean state and seasonality. As some parameters of the state-space model are unknown, we have formalized a Bayesian modeling strategy with three hierarchical levels (data, process, parameters). The possibility to further account for explanatory effects of local and/or remote co-varying processes renders the proposed state-space model ideal to statistically test specific hypotheses about the generation and propagation of systematic decadal climate prediction errors.

In the following, the methodology is described and propose one applicative examples that demonstrate how the structural decomposition and Bayesian hierarchical approach envisaged here can not only help improving the estimation of systematic climate model errors, but also better understand the relationship between different physical parameters in a region of complex dynamics such as the Tropical Atlantic.

1.1 Bayesian approach

PREFACE-MS34 provides an updated literature review of climate research studies using Bayesian hierarchical models (BHMs). BHMs use Bayes' theorem to incorporate information from different sources, including observations, physical theories and experts' knowledge. They provide a flexible framework to developing consistent inference and prediction of unknown quantities under study, which overcomes single-value estimates of uncertainty.

The BHM general formulation is based on three building blocks: the data model, the process model and the parameter model. Suppose that Z represents the data, Y represents the process and Θ represents the parameters related to the data and the process model, we then have:

(a) Data model, $[\boldsymbol{Z}|\boldsymbol{Y},\boldsymbol{\Theta}]$

(b) Process model, $[\boldsymbol{Y}|\boldsymbol{\Theta}]$

(c) Parameter model, $[\boldsymbol{\Theta}]$

where [A] is the generic notation for the probability distribution of the random quantity A. In practice, (a) defines the statistical model representing the dependence of observations on the unknown process, (b) describes the conditional probability distribution of the process on the model parameters, and (c) describes the (prior) probability distribution of the parameters, which are treated as random quantities according to the Bayesian approach.

BHMs provide estimates of the unknowns ($\boldsymbol{Y}$ and $\boldsymbol{\Theta}$) with associated uncertainty through the calculation of their posterior distribution conditioned to available observations, which is allowed by the Bayes' theorem:

$$[\boldsymbol{Y},\boldsymbol{\Theta}|\boldsymbol{Z}] = [\boldsymbol{Z}|\boldsymbol{Y}]\,[\boldsymbol{Y}|\boldsymbol{\Theta}]\,[\boldsymbol{\Theta}]\,/\,[\boldsymbol{Z}] \tag{1.1}$$

Direct evaluation of (1) to obtain the posterior distribution on the left term is often computationally intractable. This can be circumvented by generating samples from the posterior distribution through a Markov Chain Monte Carlo method.

1.2 Statistical model for the estimation of systematic decadal climate prediction errors

At the data level, the systematic error $\Delta(t)$ of the decadal climate prediction system at prediction time t is observed through differences between the predicted and the observed – or, analogously, assimilated – values ($D_j(t)$) from an ensemble of j=1,…,p hindcasts initialized at different times, according to the following model:

$$D_j(t) = \Delta(t) + \varepsilon_j(t) \tag{1.2}$$

where $\varepsilon_j(t)$ is a Gaussian white noise random error with zero mean and variance τ^2_D. We assume, for simplicity, that the observation error has the same variance in all hindcasts.

At the process level, $\Delta(t)$ is decomposed into two components: the drift/bias $\delta(t)$ and the seasonal bias $\sigma(t)$, further split into annual ($\sigma^A(t)$) and semiannual ($\sigma^{SA}(t)$). Namely:

$$\Delta(t) = \delta(t) + \sigma^A(t) + \sigma^{SA}(t) \tag{1.3}$$

The drift/bias $\delta(t)$ changes through time according to a local linear trend:

$$\delta(t) = \delta(t-1) + \beta(t-1) + \varepsilon_\delta(t) \tag{1.4}$$

$$\beta(t) = \beta(t-1) + \varepsilon_\beta(t) \tag{1.5}$$

where $\varepsilon_\delta(t)$ and $\varepsilon_\beta(t)$ are uncorrelated Gaussian white noise random errors with zero mean and variance τ^2_δ and τ^2_β, respectively. The term $\beta(t)$ is a random walk. The effect of $\varepsilon_\delta(t)$ is to allow the level of the drift/bias to shift up and down, while $\varepsilon_\beta(t)$ allows the slope to change. The larger the variances, the greater the stochastic movements in the drift/bias. If $\tau^2_\delta = \tau^2_\beta = 0$, the local linear trend collapses to a linear deterministic trend.

Seasonal terms are modeled using harmonic functions. Using monthly data with k=1 for $\sigma^A(t)$ and k=2 for $\sigma^{SA}(t)$, the k^{th} harmonic function takes the general form

$$\sigma_k(t) = \zeta^1_k\cos(2\pi kt/12) + \zeta^2_k\sin(2\pi kt/12) \tag{1.6}$$

where ζ^1_k and ζ^2_k are constants. Like the local linear trend, the seasonal term can be built up recursively, leading to the (stochastic) model for both components (see Laine et al., 2014):

$$\begin{pmatrix} \sigma_k(t) \\ \sigma^*_k(t) \end{pmatrix} = \begin{pmatrix} \cos(k2\pi/12) & \sin(k2\pi/12) \\ -\sin(k2\pi/12) & \cos(k2\pi/12) \end{pmatrix} \begin{pmatrix} \sigma_k(t\text{-}1) \\ \sigma^*_k(t\text{-}1) \end{pmatrix} + \begin{pmatrix} \varepsilon_{\sigma,k}(t) \\ \varepsilon^*_{\sigma,k}(t) \end{pmatrix} \quad k=1,2 \qquad (1.7)$$

with $\varepsilon_{\sigma,k}(t)$ and $\varepsilon^*_{\sigma,k}(t)$ uncorrelated Gaussian noise random errors with zero mean and variance τ^2_σ, and * indicating the conjugate.

The process model (3) can be easily extended to include the effect of external factors in terms of additional explanatory variables. For one dimensional explanatory variable X(*t*), the model becomes:

$$\Delta(t) = \delta(t) + \sigma^A(t) + \sigma^{SA}(t) + \gamma(t)X(t) \qquad (1.8)$$

In equation (8) we allow $\gamma(t)$ to vary according to a random walk:

$$\gamma(t) = \gamma(t\text{-}1) + \varepsilon_\gamma(t) \qquad (1.9)$$

with $\varepsilon_\gamma(t)$ Gaussian white noise random errors with zero mean and variance τ^2_γ. Time varying coefficients $\gamma(t)$ allow us to consider possible nonstationary effects of the covariate. Again, if $\tau^2_\gamma=0$, the effect collapses to be constant in time. Finally, the parameter level requires the specification of the prior distribution for the unknown parameters $\boldsymbol{\Theta} = (\tau^2_D, \tau^2_\delta, \tau^2_\beta, \tau^2_\sigma, \tau^2_\gamma)$. Setting $\tau^2_\delta=0$ leads to obtain a smoothly varying error $\delta(t)$, which then better corresponds to the drift/bias.

1.3 Implementation of the statistical model as a Dynamic Linear Model

The formulation of the statistical model in section 1.2 allows for a straightforward implementation within the dynamic linear model (DLM) framework. DLMs are based on a state-space approach, i.e., unobservable state variables are used that allow direct modeling of the process (***Y***) generating the observed data (***Z***). DLMs have the general form:

$$\boldsymbol{Z}(t) = \mathbf{F}\,\boldsymbol{Y}(t) + \mathbf{v}(t) \qquad (1.10)$$

$$\boldsymbol{Y}(t) = \mathbf{G}\,\boldsymbol{Y}(t\text{-}1) + \mathbf{w}(t) \qquad (1.11)$$

where *t* is the discrete time variable, which in a typical case represents monthly values, $\boldsymbol{Z}(t)$ is a vector of *p* observations at time *t*, $\boldsymbol{Y}(t)$ is the underlying state vector of dimension *m*, **G** is the *m*x*m* system matrix, and **F** is the *m*x*p* observation matrix. We suppose that $\mathbf{v}(t)\sim\mathbf{N}(0,\mathbf{V})$ and $\mathbf{w}(t)\sim\mathbf{N}(0,\mathbf{W})$ are the observation and model Gaussian errors, respectively, that are serially and mutually uncorrelated. In this formulation, the matrices **V** and **W** contain the model parameters $\boldsymbol{\Theta}$. If we suppose that the unknown parameters are random the DLM formulation is a BHM where (10) and (11) are typically referred to as observation equation and system equation, respectively.

In the typical case of decadal predictions consisting of multiple hindcasts, following (2) and (10), and having *p* values of D(*t*) (i.e., having *p* hindcasts) leads to the observation vector $\boldsymbol{Z}(t) = \{D_1(t), ..., D_p(t)\}'$.

In the base model without explanatory covariates, following (1.3) and accounting for annual and semi-annual seasonalities in the decadal climate prediction errors, the state vector is defined as $\boldsymbol{Y}(t) = \{\delta(t), \beta(t), \sigma^A(t), \sigma^{A}*(t), \sigma^{SA}(t), \sigma^{SA}*(t)\}'$. Therefore, the dimension of the state vector is *m*=6. If it is assumed that there is practically no systematic error during the assimilation, then it is imposed that $\boldsymbol{Y}(0)\sim\boldsymbol{N}(0,V_0)$ with V_0 very small. The observation matrix **F** is defined following (1.2) and (1.3).

The process model follows a sequential definition with conditional dependency only on the previous time step. This allows to use the Kalman filter formulas for calculating the posterior distribution (1.1) (for details see, e.g., Laine et al., 2014). Different algorithms can be used to iteratively sample from the full posterior distribution of the unknown parameters ***Θ***. In this application, we propose a slice-sampler algorithm.

Appendix 1 provides the Matlab code associated to the model described in sections 1.2 and 1.3 and associated useful data. The code is prepared to be readily used with the data provided as supplementary material of Zanchettin et al. (2017), available at https://static-content.springer.com/esm/art%3A10.1038%2Fs41598-017-13144-2/MediaObjects/41598_2017_13144_MOESM2_ESM.xls. The data are errors of spatially-averaged sea-surface temperatures for the Angola-Benguela front region, defined as the domain spanning 10°S-20°S latitude and 10°E-15°E longitude (to the coastline), for the “r1” ensemble of hindcasts initialized within the 1960-2000 period obtained from the MiKlip prototype system for decadal climate predictions (Marotzke et al., 2016). The code can be easily adapted for other use.

The code makes use of the slicesampler function from Matlab’s toolbox “Statistics and Machine Learning”, but other routines can be easily implemented. Furthermore, the code makes use of the dlmsmo routine in the dlm toolbox developed by Marko Laine (http://helios.fmi.fi/~lainema/dlm/). To use covariates in the statistical model, all lines in dlmsmo.m containing the following epression:

```
FF = [F,repmat(X(i,:),p,1)];
```

Must be commented, and substituted by the following lines:

```
if ncov>2
    FF = [F,squeeze(X(i,:,:))];
else
    FF = [F, X(i,:)'];
end
```

This allows to account for the multiple information provided at each time step by the different hindcasts.

2. Bayesian hierarchical spatial assessment of the bias in a multi-model ensemble

This section describes the statistical model used in Arisido et al. (2017a). The reader is referred to Arisido et al. (2017a) and to PREFACE-MS34 for additional details about the rationale of the approach and for illustrative examples.

Here, we only provide a few technical notes on the general approach. The model is aimed at obtaining a statistical representation of climate model biases in a multi-model ensemble to separate an overall common bias from the individual components.

Climate model bias is determined by comparing output data against observations. We let Y(s) to represent the temperature observations and $X_j(s)$ to denote the temperature simulated by the climate model j in an ensemble of Q models, at the spatial location s ∈ D for the domain D in R^2. Empirical climate model biases are then calculated as $B_j(s) = Y(s) - X_j(s)$; j = 1, ..., Q, where $B_j(s)$ denotes the bias of climate model j relative to the observations at spatial location s. For n sites in D, we observe the biases, namely $\{B_j(s_i), ..., B_j(s_n)\}$.

The Bayesian hierarchical model is formulated based on three levels: data, process, and parameters (Berliner et al., 2003, see also (1.1)). The data model captures the information given in the form of empirically measured biases, conditional on a hidden spatial bias process. The process level models the spatial structure and links the hidden spatial process to a set of parameters. In the parameter model, prior distributions are specified for the parameters. The three levels are specified in terms of probability distributions in a hierarchical structure shown in (a-c) of section 1.1 in this report.

2.1 Data model

We assume that the empirical bias $B_j(s)$ can be decomposed into two components: a spatial component $M_j(s)$ and a noise component $\varepsilon_j(s)$:

$$B_j(s) = M_j(s) + \varepsilon_j(s); \; j = 1,...,Q \qquad (2.1)$$

where $\{\varepsilon_j(s)\}$ is a Gaussian white noise with zero mean and variance $\sigma^2_{\varepsilon,j}$, and independent from the spatial component $\{M_j(s)\}$. Additionally, the noise component $\{\varepsilon_j(s)\}$ is assumed to be independent from $\{\varepsilon_k(s)\}$, for k ≠ j. Thus, conditionally on the hidden spatial process $\{M_j(s)\}$, the observed bias $B_j(s)$ has a Gaussian distribution with mean $M_j(s)$, and variance $\sigma^2_{\varepsilon,j}$ that represents the data model level.

2.2 Process model

The spatial process {M(s)}, with $M(s) = (M_1(s), ..., M_Q(s))'$ is multivariate and can be modeled in different ways (Gelfand et al. 2010). We adopt an approach based on kernel basis functions (see, e.g., Higdon, 1998) and we suppose that:

$$M_j(s) = \sum_{k=1}^{p} \beta_{j,k} w_k(s) = \mathbf{w}(s)'\beta_j, \qquad (2.2)$$

with j=1,... ,Q, and where $\mathbf{w}(s) = \{w_1(s), ..., w_p(s)\}'$ is a vector of weighting kernels, $\beta_j = (\beta_{j,1}, ..., \beta_{j,p})'$ is a vector of unknown random parameters and p << n denotes the number of components. We assume that the climate bias is an additive decomposition of a large-scale error signal and small-scale error signals including local model bias as well as local effects of, e.g., sampling of internal climate variability. The goal is to synthesize this overall common bias component, which is the same across all models in the ensemble. To this purpose we consider a random effect model (e.g., Furrer et al.

2007; Kang et al. 2012) for the random parameters β_j. More precisely, we assume that the k^{th} random parameter for the climate model j, $\beta_{j,k}$, is centered at the overall random effect α_k, namely:

$$\beta_{j,k} = \alpha_k + \nu_{j,k}, \quad j=1, ..., Q; k=1, ..., p \qquad (2.3)$$

The vector of the overall random effects $\alpha = (\alpha_1, ..., \alpha_p)'$ has multivariate Gaussian distribution $\alpha \sim Gau(\mathbf{0},\mathbf{G})$, where **G** is the non-diagonal pxp covariance matrix. The term $\nu_j =\{ \nu_{j,1}, ..., \nu_{j,p}\}'$ denotes a vector of independently distributed zero-mean Gaussian processes, $\nu_j \sim Gau(\mathbf{0}; \tau^2_j\mathbf{I}_p)$, where $\mathbf{I}_p$ is the pxp identity matrix. Centering $\beta_{j,k}$ about the overall random effects ak corresponds to our assumption that the various models share a common bias signal. Nevertheless, we expect departures of each climate model bias from the overall common bias, and this difference is reflected by the variance parameter τ^2_j. Specifically, different values of τ^2_j across the various models indicate different levels of departure from the common bias. Alternatively, similar values of τ^2_j for different models indicate that they vary similarly about the overall common bias, suggesting the contribution of each climate model in estimating the overall common bias is similar. In fact, if we impose the restriction $\tau^2_1 = \tau^2_2 = ... = \tau^2_Q$, the common overall bias corresponds to a simple average of biases from all models. Combining (2.2) and (2.3) the process model is expressed as:

$$M_j(s) = \sum_{k=1}^{p} w_k(s)\left[\alpha_k + \nu_{j,k}\right] = \mu(s) + \eta_j(s) \qquad (2.4)$$

where $\mu(s) = \Sigma^p_{k=1,\dots,p}\, w_k(s)\alpha_k$ specifies the overall common bias and $\eta_j(s) = w_k^*(s)\, \nu_{j,k}$ describes the jth model-specific features, or the departure of the jth model bias from the common overall bias. Here, we make a distinction between the weighting kernels used to describe $\mu(s)$ and η_j, i.e., w is different from w*. Since the individual components $\eta_j(s)$ aim to capture local-scale features, a larger number of kernels is required to capture this bias component compared to that necessary to describe the overall common bias, i.e., $p < p'$. Further, $\eta_j(s)$ follows the zero mean Gaussian distribution, $\eta_j(s) \sim Gau(\mathbf{0}, \tau^2_j\mathbf{w}^*(s)\mathbf{w}^*(s)')$. In other words, the model suggests that the spatial process $M_j(s)$ is decomposed into an overall common component $\mu(s)$ and an individual component $\eta_j(s)$.

In the following, the methodology is described and propose one applicative examples that demonstrate how the structural decomposition and Bayesian hierarchical approach envisaged here can not only help improving the estimation of systematic climate model errors, but also better understand the relationship between different physical parameters in a region of complex dynamics such as the Tropical Atlantic.

2.3 Model implementation

The model is implemented in the software R, making use of the “OpenBUGS” package. The software and the data are available as supplement of Arisido et al. (2017a).

Appendix 2 contains the relevant information (code and test data) to run the model.

3. Spatio-temporal assessment of climate model bias: a Bayesian approach

This section describes the statistical model used in Arisido et al. (2017b), of which it is an excerpt.

Climate model error (hereafter referred to as deviation) is determined by comparing output data simulated from the climate models against observations. We let $Y_t(\mathbf{s})$ and $X_t(\mathbf{s})$ to represent the observed and the simulated value of a certain geophysical quantity, respectively, at spatial location **s**, with $\mathbf{s} \in \{s1,\ldots sn\}$ in a region $\mathscr{D} \in R^2$ and time $t \in \{1,\ldots,T\}$. We derive the spatio-temporal climate model deviation as

$$D_t(\mathbf{s}) = Y_t(\mathbf{s}) - X_t(\mathbf{s}),\ t = 1,\ldots,T \qquad (3.1)$$

where $D_t(\mathbf{s})$ denotes the deviation of the simulated value relative to the observations at spatial location **s** and time t. For n spatial locations in $\mathscr{D}$, we observe the deviations $D_t(s1),\ldots,D_t(sn)$ for the time t. From the spatio-temporal deviation $D_t(\mathbf{s})$, one can calculate the empirical systematic bias $B(\mathbf{s})$ as $B(\mathbf{s}) = \sum_{t=1:T} D_t(\mathbf{s})/T$. We remark that, generally, statistical analysis of climate model deviations can be affected by the spatial misalignment between observations and model output since the model output and the observations are on different grids. In PREFACE-WP10 we tackled this issue by interpolating the model output data on the regular observational grid to ensure that $Y_t(\mathbf{s})$ and $X_t(\mathbf{s})$ are aligned on the same grid (see PREFACE-D10.2).

The aim now is to formulate a statistical model to quantify and characterize climate model errors accounting for their inherent spatial and temporal dependencies. We specify the model in the Bayesian hierarchical framework based on three levels: data, process, and parameters (see (a-c) and PREFACE D10.2). So, our model specification is structured with (1) a data model describing the information given in the form of the empirically observed deviation, conditional on unobserved spatio-temporal deviation process under investigation; (2) the unobserved process featuring spatio-temporal characters described using a set of parameters and (3) the parameters that appear in the first two levels, and specify their prior beliefs according to Bayesian reasoning.

3. 1 Data model

The idea is that in the evaluation of the systematic bias $B(\mathbf{s})$ the local spatio-temporal effects should be filtered out. To model the deviation, we assume that the observed deviation $D_t(\mathbf{s})$ can be decomposed into two components:

$$D_t(\mathbf{s}) = M_t(\mathbf{s}) + \varepsilon_t(\mathbf{s}) \qquad (3.2)$$

where $M_t(\mathbf{s})$ is a spatio-temporal Gaussian random field and $\varepsilon_t(\mathbf{s})$ is a temporally and spatially uncorrelated zero mean Gaussian noise with variance s_t^2. Note that the model is allowed to take into account for the heterogeneity in time. We assume that the noise component $\varepsilon_t(\mathbf{s})$ is independent of the deviation process $M_t(\mathbf{s})$. In practice, we convey into the process $M_t(\mathbf{s})$ all smoothed spatio-temporal components that actually are blurred by the noise term. We further assume that the observed deviation $D_t(\mathbf{s})$ is conditionally independent in time given $M_t(\mathbf{s})$. Such assumptions lead to the data model in the form

$$[D_1(\mathbf{s}),\ldots,D_T(\mathbf{s})|M_1(\mathbf{s}),\ldots,M_T(\mathbf{s}),\sigma_1^2,\ldots,\sigma_T^2] = \prod_{t=1}^{T}[D_t(\mathbf{s})|M_t(\mathbf{s}),\sigma_t^2] \qquad (3.3)$$

where [A] denotes the generic notation for the probability distribution of the random quantity A. Accordingly [A|B] is the conditional distribution given B.

3.2 Process model

The process model characterizes the spatio-temporal deviation process $M_t(\mathbf{s})$. Once we determine $M_t(\mathbf{s})$, an important interest will be to estimate the more appropriate time-invariant systematic bias $\tilde{B}(\mathbf{s})$ for the study period as an average of $M_t(\mathbf{s})$, i.e., $\tilde{B}(\mathbf{s}) = \sum_{t=1:T} D_t(\mathbf{s})/T$. The spatio-temporal process $M_t(\mathbf{s})$ is driven by a large-scale spatial component that changes stochastically but smoothly in time and a site-specific component. The spatial large-scale component at time t is represented by a linear combination of p spatial kernel functions $\{\Psi_k(\mathbf{s}) : k = 1,\ldots, p\}$ as in Higdon (1998), i.e., $\sum_{k=1:p} \Psi_k(\mathbf{s})\, \beta_{t,k}$ where $\beta_{t,k}$ is the coefficient parameter for kernel k. The whole formulation is given by

$$M_t(\mathbf{s}) = \Psi(\mathbf{s})'\, \beta_t + \nu_t(\mathbf{s}) \qquad (3.4)$$

$$\beta_t = \beta_{t-1} + \omega_t \qquad (3.5)$$

$$\nu_t(\mathbf{s}) = \nu_{t-1}(\mathbf{s}) + \delta_t(\mathbf{s}) \qquad (3.6)$$

where $\Psi(\mathbf{s}) = \{\Psi_1(\mathbf{s}), \ldots, \Psi_p(\mathbf{s})\}$ and $\beta_t = (\beta_{t,1}, \ldots, \beta_{t,p})'$. The number of kernels p is chosen to be much less than the number of spatial data points n. The choice of the kernels is further discussed in section 3.3. Equation (3.5) states that the p x 1 vector of the linear coefficients β_t changes according to a random walk process, where the evolution error ω_t is assumed as an independently and identically distributed zero mean Gaussian process with variance-covariance matrix Σ_ω. Then, equation (3.6) defines the site specific component $\nu_t(\mathbf{s})$ in order to account for the underlying spatial correlation, capturing its Markovian dependence in time. More specifically, $\delta_t(\mathbf{s})$ follows a zero mean spatial Gaussian process with covariance function C_t , which is specified as $C_t(\mathbf{s}, \mathbf{s'}; \theta_t) = \tau_t^2\, \rho(\mathbf{s}, \mathbf{s'}; \phi_t)$, where $\theta_t = \{\tau_t^2, \phi_t\}$ and $\rho(.,\phi_t)$ is a correlation function with ϕ controlling the correlation decay and τ_t^2 representing the spatial variance. Any valid spatial correlation function can be used to define $\rho(.,\phi_t)$ (e.g., see Cressie, 1993). Here we use the exponential function, i.e., $C_t(\mathbf{s}, \mathbf{s'}; \theta_t) = \tau_t^2 \exp(-\phi_t \,\|\, \mathbf{s}\text{-}\mathbf{s'} \,\|)$, where **s**-**s'** is the Euclidean distance between locations **s** and **s'**. Further, for each time point t, ω_t is uncorrelated with $\varepsilon_t(\mathbf{s})$. The different levels of the Bayesian hierarchical approach discussed above can be formulated within a state-space form (Gelfand et al. 2005; Durbin and Koopman 2012). That is, combining the data model (3.2) and the process models (3.4)-(3.6) yields

$$D_t(\mathbf{s}) = \Psi(\mathbf{s})'\, \beta_t + \nu_t(\mathbf{s}) + \varepsilon_t(\mathbf{s}) \qquad (3.7)$$

$$\beta_t = \beta_{t-1} + \omega_t \qquad (3.8)$$

$$\nu_t(\mathbf{s}) = \nu_{t-1}(\mathbf{s}) + \delta_t(\mathbf{s}) \qquad (3.9)$$

where (3.7) is the measurement equation, and (3.8,3.9) are the transition equations. While (3.7) is similar to the measurement equation of the standard state space model, we recognize that assuming a random walk process in transition equations is a simplification from the more general specification (as provided in, e.g., West and Harrison, 1997). Nonetheless the random walk is chosen to provide adequate flexibility for computation and eases the interpretation (e.g., Finley et al. 2012).

3. 3 Parameter model

We complete the model specification by assigning prior probability distributions for the initial conditions $\{\beta_0, \nu_0(\mathbf{s})\}$ and the model parameters $\{\Sigma_\omega, (\sigma_1^2, \theta_1), \ldots, (\sigma_T^2, \theta_T)\}$.

Prior distributions for these parameters are generally taken to be non-informative. For the initial conditions, we specify Gaussian priors in the form $\beta_0 \sim N(\mu_{\beta 0}, \Sigma_{\beta 0})$ where $\mu_{\beta 0}$ is a vector of length p and $\Sigma_{\beta 0}$ is a p x p covariance matrix, and $\nu_0(\mathbf{s})=0$. Recalling $\theta_t = \{\tau_t^2, \phi_t\}$ for the measurement error variance σ_t^2 and the spatial variance τ_t^2 we assign the Inverse-Gamma priors $\sigma_t^2 \sim IG(a_1, b_1)$ and $\tau_t^2 \sim IG(a_2, b_2)$ for each t, where IG(a, b) denotes the inverse gamma distribution with shape parameter a and scale parameter b. Here $\{\mu_{\beta 0}, \Sigma_{\beta 0}, a_1, b_1, a_1, b_2\}$ are called hyperparameters in the Bayesian context, and their values could either be chosen or could be assigned another priors (see, e.g., Gelman 2006). For the spatial decay parameter ϕ_t of the exponential spatial correlation function, we suggest to assign the uniform prior in the form $\phi_t \sim U(0.001;0.04)$, which corresponds to the support ranges from 100 to 4000 km, for regions of the size of the tropical Atlantic. In fact, for current reanalysis products, since the maximum distance between any two locations in this region is O(3000 km), the specified support well covers the full extent of the spatial domain. For the p x p evolution matrix Σ_ω, we assume the inverse-Wishart prior probability distribution, $\Sigma_\omega \sim IW(p+1, \mathbf{I}_p)$, with the degrees of freedom parameter taking the value p+1 and the scale parameter being the p x p identity matrix $\mathbf{I}_p$, as we assume independence between the elements of the coefficient vector β_t . These choices correspond to relatively non-informative priors, and our sensitivity analysis indicated that the results are not substantially sensitive to these choices.

3.4 Model implementation

First we discuss the choice of the spatial kernel vector $\psi(\mathbf{s})$. Several types of kernel functions have been suggested, including Gaussian kernels (Stroud et al. 2001), harmonic functions (e.g., Furrer et al. 2007) and bisquare functions (Kang et al. 2012). In the proposed model we have considered a Gaussian kernel specified as

$$\psi_k(\mathbf{s}) = \exp\{ -(s - c_k)' \Sigma^{-1} (s - c_k)/2\}, k = 1, \dots , p \qquad (3.10)$$

where c_k denotes the center of the kernel and Σ determines the shape. The number of kernels p, their locations and shapes must be chosen. These choices are often based on the presence of prior information such as smoothness and spatial dependence related to the spatial process under study (Stroud et al. 2001). If we choose spherically shaped kernels, i.e., $\Sigma = kI_2$ on R^2 and $k > 0$, and the centers belong to a regular grid over an unbounded domain, the resulting spatial process approximates a covariance function of a stationary isotropic process when the number of kernels p is very large. Alternatively, a geometrically anisotropic process may be obtained if we choose non-spherical Gaussian kernels. One way to assess the shape of S is to perform variogram analyses for different directions (see, e.g., Cressie 1993). Figure 3.1a shows an example of such analysis for the region of the tropical Atlantic. The variogram suggests that isotropy is a plausible assumption for $M_t(\mathbf{s})$ at small distances since the patterns are not largely different from each other. If the analysis is confirmed at several time points suggests, equally-spaced Gaussian kernels can be used (see Figure 3.1b) We recommend to always perform sensitivity analyses for different choices of p.

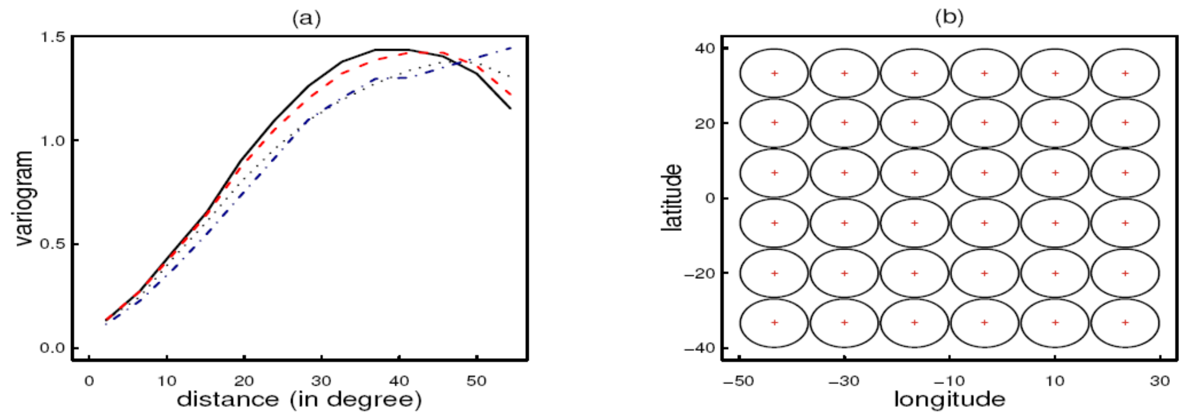

Figure 3.1 - Example of kernel specification for gridded data for the region of the tropical Atlantic. (a) Empirical variogram for a given time t for four different directions (black solid: 0, red dashed: 45, gray dotted 90, blue dashed 135). The variogram was analyzed using the robust estimator by Cressie (1993) and indicates no anisotropy. (b) Spherically-shaped 36 equally-spaced Gaussian kernels. Red crosses indicate the centers of the kernels.

Once a reasonable choice of $\psi(\mathbf{s})$ is made, the model can be implemented in the Bayesian context. For parameter estimation and inference, we seek to obtain the posterior distribution of the unknown parameters $\{\beta_0, \Sigma_\omega, (\beta_1, \sigma_1^2, \theta_1), \ldots, (\beta_T, \sigma_T^2, \theta_T)\}$. For a particular location **s**, the posterior distribution can be given in the form

$$[\beta_0, \beta_{1:T}, \Sigma_\omega, \sigma_1^2, \theta, \ldots, \sigma_T^2, \theta_T | D_{1:T}(\mathbf{s})] \propto \prod_{t=1}^{T}[D_t(\mathbf{s})|\beta_t, \sigma_t^2] \times [\beta_0] \times \prod_{t=1}^{T}[\beta_t|\beta_{t-1}, \Sigma_\omega] \times \prod_{t=1}^{T}[\sigma_t^2] \times \prod_{t=1}^{T}[\theta_t] \times [\Sigma_\omega] \quad 3.11$$

with notations as in Cressie and Wikle (2015). Clearly, the normalizing constant for (3.11) cannot be found analytically. So, we use the Markov Chain Monte Carlo (MCMC) method with Gibbs sampler and random walk Metropolis steps (Robert and Casella 2013). For the random walk Metropolis step, a multivariate normal (same dimension as the number of model parameters) proposal distribution is used. Inspection of graphical tools of the simulation history can be used to assess convergence.

The model is implemented in the software R using the spBayes package (Finley et al. 2015). Code and data will be made available as supplement of Arisido et al. (2017b). **Appendix 3 contains the relevant code to run the model**.

The computation time depends mainly on the size of the kernel vectors, the spatial coverage and the number of time points. For example, in the main analysis of Arisido et al. (2017b), the model was used to analyze the spatio-temporal bias evolution for an ensemble-mean of six historical full-forcing climate simulations contributing to CMIP5, for the period 1948-2005 and for a spatial domain covering the tropical Atlantic and bordering regions. There, a Gaussian kernel vector with length 36, a regular grid of 33 x 33 = 1089 sites and T = 58 years were considered. The computations took about 40 hours on a 64-bit Unix workstation Intel Xeon 2.60 GHz.

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Appendix 1

Matlab-code and data to perform structural decomposition of systematic climate model errors described in section 1 of this deliverable. Note that DATASET1.xls is available as supplement to the open access publication Zanchettin et al. (2017)

run_dlm_index.m

```
function run_dlm_index
% This function performs dlm calculation and mcmc for index time series.
% It requires the dlmsmo.m routine by Marko Laine and the slicesample.m
% routine of Matlab's "statistics and machine learning" toolbox.
% See PREFACE-D10.3 for further details

% ========================================================================
% 1 LOAD DATA and DEFINE MODEL'S PARAMETERS
% ========================================================================
% set all model parametersin the struct variable modelpar:
modelpar.nmcmc = 30000;  % number of mcmc iterations
modelpar.nburn = 10000;  % number of burn-out iterations
modelpar.nthin=10; % steps for thinning
modelpar.nsamples = 500;  % number of samples to retain. Must be <=nmcmc
modelpar.covname={'tauu'}; % covariate names, data are assumed to be in {covname}.dat
modelpar.covlead=[0 0]; % how many month the covariate leads the main variable

outfileroot=['test1'];

% load data matrix (D in equation 3 of D10.3)
% D must have the form [n x p], with n the number of timesteps and p the
% number of hincasts.
D = xlsread('DATASET1.xls'); % change to read other data
[n,p]=size(D); % following notation by dlmtoolbox

% Load covariate data, if requested
ncov=length(modelpar.covname);
COVINDEX=[]; % initialize covariates' data matrix
if ncov>0,
  for icov=1:ncov
     covdata=load([modelpar.covname{icov} '.dat']);
     COVINDEX=cat(3,COVINDEX,covdata);
  end
  % check that size of D and COVINDEX match:
  if size(COVINDEX,1)~=n || size(COVINDEX,2)~=p
     error('dimension mismatch: check covariate data')
  end
end

% ========================================================================
% 2. DEFINE AND RUN SPACE-STATE MODEL
% ========================================================================
% generate matrices for observation and system equations
% system equation:
[G,winds] = createG(ncov);
m = size(G,1); % following notation by dlmtoolbox
% observation/measurements equation:
F = createF(p);

% Initialize uncertainties:
s = 0.5.*ones(size(D)); % obs uncertainty, set by default at 0.5 for all points
wdiag = 0.5.*ones(size(winds));
wdiag(~winds)=0; % put zero where there is not estimation of error
W = zeros(m);
for i=1:max(winds)
   j=find(winds==i);
   for jj=1:length(j)
      W(j(jj),j(jj)) = 0.25;
   end
end
V = s;

% Define initial state x0 and initial state uncertainty covariance
x0 = zeros(size(G,2),1);   % assume all bias components are 0 at t=0.
C0 = eye(m)*s(1)^2; % define here how large the uncertainty on state at t=0 should be
```

```
% note s and V are in st. dev. following dlmsmo, so variance is 0.025 as in Zanchettin et al. (2017)

% check dimensions of inputs to dlmsmo:
% n = length of time series
% p = number of time series from measurements
% m = dimension of state space
if size(D,1)~=n || size(D,2)~=p
   error('Wrong dimension of measurement data matrix D')
elseif size(F,1)~=p || size(F,2)~=m-ncov
   error('Wrong dimension of measurement operator matrix F')
elseif size(x0,1)~=m || size(x0,2)~=1
   error('Wrong dimension of initial state vector x0')
elseif size(G,1)~=m || size(G,2)~=m
   error('Wrong dimension of system evolution matrix G')
elseif ~isempty(C0) && ( size(C0,1)~=m || size(C0,2)~=m )
   error('Wrong dimension of initial state uncertainty covariance matrix C0')
end

% call dlm time series model:
dlm = dlmsmo(D,F,V,x0,G,W,C0,COVINDEX);

% =========================================================================
% 3. SET AND RUN MCMC, THEN SAMPLE
% =========================================================================
% define target function (i.e., posterior) for the MCMC:
if ncov==0
   postpdf = @(prm) lognpdf(prm(1),0,1)+lognpdf(prm(2),0,1)+lognpdf(prm(3),0,1) ... +lognpdf(prm(4),0,1) ... % priors for V (one) and W (two)
           -0.5*fitlikelihood(prm,dlm,winds); % likelihood
else
   str=['postpdf = @(prm) lognpdf(prm(1),0,1)+lognpdf(prm(2),0,1)+lognpdf(prm(3),0,1)']; % priors for V (one) and W (two)
   for i=1:ncov
      str = [str '+lognpdf(prm(' num2str(i+3) '),0,1)']; % covariates
   end
   str = [str '-0.5*fitlikelihood(prm,dlm,winds);']; % likelihood
   eval(str)
end
% set starting values for all parameters:
prmi=[zeros(max(winds)+1,1); -0.5*dlm.lik];

fprintf(['\n           Running MCMC'])
prm = slicesample(prmi,modelpar.nmcmc,'logpdf',postpdf,'thin',modelpar.nthin,'burnin',modelpar.nburn);

% save chain in intellegible way for dlmsmo (logpdf are used in the mcmc):
prm(:,1:end-1)=exp(prm(:,2:end-1)); % except likelihood

% initialize samples from the MCMC:
xsample=NaN.*ones(m,n,modelpar.nsamples);
% define which steps of the chain taken for the sampling:
isamples=randperm(size(prm,1),modelpar.nsamples);

RESID=[]; % residuals
for i=1:modelpar.nsamples
   isi=isamples(i);
   V = ones(n,p).*prm(isi,1);
   W = zeros(m);
   for wi=1:max(winds)
      j=find(winds==wi);
      for jj=1:length(j)
         W(j(jj),j(jj)) =  prm(isi,wi+1);
      end
   end
   o = dlmsmo(dlm.y,dlm.F,V,x0,G,W,C0,dlm.XX,1);
   RESID=[RESID o.resid];
   xsample(:,:,i)=o.xr;
end

% save relevant output
o.D=D;
o.resid=RESID;
o.chain=prm;
o.modelpar=modelpar;

str=['save dlm_' outfileroot '.mat o']; eval(str)
str=['save xsamples_' outfileroot '.mat xsample']; eval(str)


% =========================================================================
```

```
% ====     AUXILIARY FUNCTIONS  ========================================
% ======================================================================
function [G,winds] = createG(ncov)
% creates the system matrix G for the state-space model:
% D(t) = F*X(t) + em(t) (measurement)
% X(t) = G*X(t-1) + es(t) (system)
% em(t) ~ N(0,V)
% es(t) ~ N(0,W)

% In this case:
% X(t) = [B(t); Tr(t) SB_h1; SB_h2; RB(t); gamma(t)]
% where: B(t) = local model bias at time t
%       Tr(t) = trend component
%       SB_h1/2 = first/second harmonic of bias in 12-m seasonal cycle
%       gamma(t) = covariate(s)
% This can be easily adapted to other needs.

G = [ 1.0000    1.0000        0        0       0      0   ; ...
        0     1.0000        0        0       0      0   ; ...
        0        0       0.86603    0.5000    0      0   ; ...
        0        0       -0.5000    0.86603   0      0   ; ...
        0        0          0        0      0.5   0.86603 ; ...
        0        0          0        0  -0.86603    0.5   ; ...
        ];

% specify std for normal priors of wdiag (following dlmsmo.m):
winds=[0 1 2 2 2 2];

% Add entries in case covariates are included (for gamma(t)):
if ncov>0
   for i=1:ncov
      G=cat(2,G,zeros(size(G,1),1));
      G=cat(1,G,zeros(1,size(G,2)));
      G(end,end)=1;
      winds=[winds max(winds)+1];
   end
end

% ======================================================================
function F = createF(p)
% creates an observation matrix F for the state-space model:
% D(t) = F*X(t) + em(t) (measurement)
% X(t) = G*X(t-1) + es(t) (system)
% em(t) ~ N(0,V)
% es(t) ~ N(0,W)

% In this case:
% X(t) = [B(t); Tr(t) SB_h1; SB_h2; RB(t); gamma(t)]
% where: B(t) = local model bias at time t
%       Tr(t) = trend component
%       SB_h1/2 = first/second harmonic of bias in 12-m seasonal cycle
%       gamma(t) = covariate(s)
% This can be easily adapted to other needs.

% INPUT
% p: number of ensemble members
% M: number of initialized ensembles
% ----------------------------------------------------------------------
% F must have obviously the following dimensions: N and size(G,1)
% Must be constructed according to G.
% Entries for covariates are included in dlmsmo.m (see PREFACE-D10.3)

F = repmat([1 0 1 0 1 0],p,1);
% ======================================================================
```

Test data for covariate tauu. Copy-paste the text below in a text file called “tauu.dat” and place it in the same folder as “run_dlm_index.m”.

tauu.dat

% empirical hindcast errors in monthly mean spatially-averaged "Surface Downward Eastward Wind Stress" in the Angola-Benguela front region used in Zanchettin et al., Sc. Rep.,
% columns are different hindcasts corresponding to the ""r1"" realization for each initialization year between 1960 and 2000
% rows are different hindcast times t=1,...,120. Units: Pa
2.2144265e-02 2.7161762e-03 4.4737756e-02 -5.2170381e-03 -6.6617578e-03 2.7453765e-02 6.6519380e-03 3.2988109e-02 3.0293569e-02 6.6565681e-02 2.9026054e-02 5.2816980e-02 3.2442596e-02 1.9907273e-02 3.8527027e-02 3.2906631e-02 4.4050120e-02
4.5354180e-02 6.0759671e-02 6.2541861e-02 5.3662408e-02 6.0858969e-02 6.1472379e-02 3.2590747e-02 3.2696821e-02 4.0734865e-02 1.7947726e-02 4.0013961e-02 8.7437779e-03 3.9067980e-02 2.9781356e-02 6.1705491e-02 1.7060146e-02 2.6515290e-02
2.0192906e-02 2.6751462e-02 4.8431877e-02 1.4491133e-02 3.6138855e-02 2.1058645e-02 3.8957410e-02
-1.0759834e-02 -1.6762901e-02 3.3049121e-02 -5.0033499e-02 2.4427995e-03 -5.9766695e-03 -3.1003617e-03 -1.8580768e-02 5.0573368e-03 1.3638651e-02 4.3163307e-02 -7.5942352e-03 3.1943385e-02 2.3005642e-03 2.2548908e-02 8.7210201e-03 -2.8557941e-02
1.3589919e-02 1.4899373e-02 4.6206661e-02 1.6439542e-02 -3.6174357e-03 4.3036386e-02 3.9680302e-04 -8.6673200e-03 7.9686977e-03 3.5882592e-03 1.0663349e-02 -1.1656806e-02 2.0730760e-02 1.3109338e-02 1.3236426e-02 -2.3689549e-02 2.2442009e-02
1.9003730e-02 1.0989502e-02 -4.4355914e-04 -1.8583830e-02 5.2118801e-02 -1.2059446e-02 4.6015903e-03
-2.7734622e-02 -3.3092911e-02 -1.9459222e-03 -1.3172235e-02 3.8361177e-05 -1.2485172e-02 -2.9647628e-02 -1.8441516e-02 -1.1975691e-03 6.0710507e-03 2.7073899e-02 -2.4313728e-02 2.5288690e-02 2.2228894e-02 -1.2458184e-02 5.2859529e-03 8.4458615e-03
-5.3564981e-03 1.1890672e-02 1.1292717e-02 -1.2007794e-02 1.0973047e-02 6.8155181e-03 4.3823444e-03 3.4647137e-03 -7.7338666e-03 -8.9221820e-04 -1.5286809e-02 1.0564020e-02 -2.1965679e-02 -1.0161719e-02 -1.7864984e-02 -9.1766305e-03 -1.6191428e-02
1.3581669e-02 -1.1185307e-02 -7.9374313e-03 -9.9591874e-03 1.4249420e-02 -1.9981463e-02 -4.3143881e-03
-1.9828674e-02 -4.7385121e-02 -8.7832762e-03 2.3821552e-03 -4.2364254e-02 7.4263886e-03 -2.2899474e-02 -3.6476607e-02 6.7882705e-04 -6.6609131e-03 -6.8211593e-03 -1.2021161e-02 2.0017711e-02 -1.6663019e-03 6.0603227e-03 2.6415847e-03 1.2111377e-02
-7.0235962e-03 -1.5634714e-02 2.1469258e-02 5.4187691e-02 -2.7322841e-02 -1.0766504e-02 2.0799499e-02 -3.6423442e-02 -5.4799840e-03 -1.6972734e-02 -2.0119441e-02 -9.7474721e-03 -5.4124357e-02 3.2952875e-03 -2.5132008e-02 -4.2544928e-03 -2.4329798e-02
-6.3176453e-04 5.9997924e-03 9.8503702e-03 3.2813076e-02 6.9102459e-03 1.2519429e-02 -2.6806363e-02
-3.6185908e-02 2.6875231e-03 -1.1035267e-02 -3.0732285e-03 -1.0370540e-02 7.2785914e-03 -5.0677961e-02 -1.5384050e-02 -4.5735519e-02 -1.6989550e-02 -3.2755435e-03 -6.2408671e-04 -5.1953191e-02 1.4923736e-02 -7.6699443e-04 -1.6231321e-02 1.6605539e-
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-2.6995860e-02 2.7384618e-02 -4.9133753e-02 -8.9974329e-03 -1.0359142e-02 -1.8251186e-02 -2.8057178e-02 -2.8266963e-02 -2.3793146e-02 -4.3035135e-02 -7.0029749e-03 -1.3297141e-02 -2.5540801e-02 7.8156590e-06 9.8505616e-04 1.5977729e-02 -2.0801447e-02
2.3309618e-02 -2.1942085e-02 -2.7307615e-02 4.6352707e-03 6.6055432e-03 -2.0107565e-02 -3.7705325e-02
-1.1961643e-02 -2.1874450e-02 -3.1050249e-02 7.0796758e-03 -1.5840031e-02 -7.0330419e-02 -3.2328572e-02 -1.1770725e-02 1.3000876e-02 -5.5371895e-03 -1.6671084e-02 -1.3995841e-03 4.4899397e-03 -5.0624760e-02 1.3873167e-02 -3.0541256e-02 -4.1198261e-
02 -8.6430423e-03 -1.5422855e-02 -1.9855773e-02 -1.8177409e-02 -3.4432260e-02 -1.6951762e-02 -7.6286756e-03 2.3692958e-03 -2.9975038e-02 -3.5761643e-02 -6.4327591e-02 3.5502426e-03 -4.2766333e-06 8.8565052e-03 3.1374246e-03 -3.3734236e-02 -
9.1621689e-03 -1.1568293e-02 -2.4535632e-02 -2.4026070e-02 -4.9105148e-02 -1.2359168e-02 -3.7016636e-02 -4.0065795e-03
-4.8489012e-02 -4.0513112e-02 -7.1755130e-02 -2.9423201e-02 -4.8461098e-02 -5.9968182e-02 -3.3805672e-02 -3.7814129e-02 3.3281669e-03 -5.6491727e-02 -1.4963739e-02 -7.4960254e-02 1.2094686e-02 -3.2903232e-02 1.4523230e-02 -2.9393107e-02 -3.5071433e-
02 -2.8501960e-02 -5.1543416e-02 -4.0239738e-02 -3.9419668e-02 -3.4234013e-02 -1.3440438e-03 1.2264684e-02 -3.6034681e-02 -5.1915038e-02 -3.6008124e-02 -1.8369351e-02 -2.5379458e-02 -4.0240947e-02 -2.0001445e-02 -3.0347848e-02 -9.7751542e-02 -
1.6100027e-02 -3.6246970e-02 -1.7671660e-02 -2.4422927e-02 1.6971231e-03 -1.8618550e-02 -1.1580598e-02 -2.8167926e-03
-1.7550332e-02 -8.4363199e-02 -5.0595387e-02 9.1442894e-03 -1.8042479e-02 -2.1275658e-02 -2.1571167e-02 1.8542577e-03 -4.5152768e-02 -3.5625283e-02 -5.7969615e-02 -7.0207754e-02 -3.3099389e-02 -3.4676340e-02 -1.7631529e-02 -2.8318630e-02 -5.5900004e-
02 -3.8904184e-03 -3.4532723e-02 -2.7751205e-02 -7.1507756e-02 -2.8862539e-02 -3.7579794e-02 -3.4491823e-02 -2.0027638e-02 -4.4632615e-02 -5.3994002e-02 -9.2866722e-02 -3.3584118e-02 -2.0084654e-02 -2.7903151e-03 -7.6891967e-02 -7.0669601e-02 -
7.0678871e-02 -7.1426224e-02 -2.4352784e-02 -8.0520032e-02 -5.3312301e-02 -1.6109462e-02 -3.6429001e-02 -2.6428729e-02
-1.5451820e-02 -2.5386451e-03 -8.2593963e-02 -3.6245486e-02 -8.3899897e-03 -3.8933454e-02 -7.7512970e-03 -6.1918693e-03 -6.2298013e-02 -8.3108295e-03 -2.4209721e-02 8.8135782e-03 -1.3202711e-02 -4.2676149e-02 -1.3211379e-02 -3.0861948e-02 -2.5898098e-
02 -5.8687128e-02 -3.3516545e-02 -1.9671914e-02 -3.2267861e-02 -4.2224209e-02 -5.0803557e-02 5.0504917e-03 -3.9192353e-02 -3.1026189e-02 -4.6352572e-02 -4.0970353e-02 -4.7162034e-02 -5.1087815e-02 -1.7626365e-02 -3.9035387e-02 -4.0779185e-02 -
5.8568478e-02 -2.5560571e-02 -1.7645393e-02 -8.8030072e-03 -3.1833140e-02 -3.0346501e-02 -1.2814184e-02 -5.6690747e-02
2.7792282e-02 4.4448525e-03 -2.0534210e-02 4.9798908e-03 -8.3407671e-03 7.8446781e-03 -1.2663275e-02 2.2154449e-02 -3.2890053e-02 -6.1549656e-03 1.7791262e-02 -1.1287047e-02 -1.4586808e-02 -2.6638712e-02 9.5734242e-03 -1.3607494e-02 -2.3279690e-02
9.6282214e-03 -5.4043260e-02 -1.0751317e-02 1.4537107e-02 -2.9165646e-02 -3.2116283e-02 -1.7405991e-02 -1.3604604e-02 -3.8834993e-02 -3.3185685e-02 -3.4023066e-02 -2.2785459e-02 2.5012344e-03 -4.3564162e-02 -2.8076350e-02 -2.4557436e-02 -1.7370477e-02
-2.0845532e-03 -1.4668996e-02 4.6487963e-03 -2.4287166e-02 -2.1421418e-02 -1.8242418e-02 -4.5456568e-02
-1.2715667e-02 -8.7857321e-03 4.4149216e-03 1.8024262e-02 -1.3142344e-02 1.2204997e-02 1.7670915e-04 1.4414977e-02 2.8887144e-03 1.4003133e-02 -3.1748320e-02 4.4461712e-04 -3.6532138e-02 -2.3544312e-02 -1.5318410e-02 -3.3617057e-03 -1.2580147e-02
5.8027320e-03 -1.7692470e-02 -1.9784123e-03 -6.8933871e-03 1.2273416e-03 -9.9278670e-03 -5.6059380e-02 -1.2606882e-02 -4.6874173e-03 5.8103092e-03 -3.7406353e-02 -7.2047859e-03 2.3713261e-03 -4.7567459e-02 -3.8326203e-02 -6.5507460e-03 -8.7151341e-03
9.2605390e-03 -4.9097538e-03 5.0981175e-03 -3.1077486e-02 -2.6743917e-02 -1.2912601e-03 -3.6539211e-02
-4.8520304e-03 -1.1274217e-02 -2.5593316e-02 -6.8314373e-04 -6.5209977e-03 -9.9187344e-04 -2.0223649e-02 -1.3886876e-02 -2.6994580e-02 8.1178844e-03 -4.0162504e-03 2.3665842e-02 -1.0474285e-02 -2.6113600e-02 -8.8133402e-03 -3.6009151e-02 -7.6540522e-
03 -2.8574357e-02 1.8144064e-03 -3.0102510e-02 -7.6553822e-03 -3.4124479e-03 -2.8390177e-02 -1.1468634e-02 4.7438778e-03 -3.4445968e-02 -2.0121373e-03 -3.8686842e-03 3.0091777e-03 1.0223871e-02 -3.1992335e-02 -8.8647939e-03 -3.2779267e-02 -
1.1965763e-02 -2.2990458e-02 -1.8729821e-02 -7.4400119e-03 -4.2035799e-02 1.2506422e-02 4.5993254e-03 -3.1102572e-02

Appendix 2

R-code and data to perform Bayesian hierarchical spatial assessment of the bias in a multi-model ensemble described in section 2 of this deliverable.

The file 'R-code.txt' is the main R code, which uses the 'R2OpenBUGS' package to call a BUGS model and summarize results in R. The file provides indications about the necessary installation of software packages. The code in 'R-code.txt' should be copy/pasted to the R shell to run the codes.

The file 'Model-name.txt' contains the model written in OpenBUGS. The extension must be '.txt' and should be placed in the current working directory.

The data labelled 'Data-bias' are the working data provided to run the code. The data is a matrix with dimension n x J where n = 525 is the number of grid locations and j = 6 is the number of climate models considered in the paper.

Files 'Lon' and 'Lat' are the geographic coordinates associated to longitude and latitude, respectively.

R-code.txt:

```
###################################
##set your working directory here
###################################

setwd("/Users/maeregu/Dropbox/supplemental")
getwd()
dir()

###################################
###  Install R Packages here
###################################

library(geoR)
library(fields)
library(maps)
library(sp)
library(raster)
library(grid)
library(R2OpenBUGS)
library("ellipse")
library("MBA")
library(sbgcop)


#############################################
## Read the data from your working directory
############################################


B<-as.matrix(read.table(file="/Users/maeregu/Dropbox/supplemental/Data-bias.txt",sep=",",header = T))
colnames(B, do.NULL = TRUE)
colnames(B) <- c("CCSM4","BCC","IPSL","MPI","GISS","MIROK")
dim(B)
B<-as.data.frame(B)
B.names <- names(B)

Lon<-read.table(file="/Users/maeregu/Dropbox/supplemental/Lon.txt",sep=",",header = F)[,1]
Lat<-read.table(file="/Users/maeregu/Dropbox/supplemental/Lat.txt",sep=",",header = F)[,1]

##########################################
## Define grids of spatial location covering
## Tropical Atlantic
##########################################

coords<-as.matrix(expand.grid(Lon,Lat))

########################################################
##### Summarize the empirical biases from the six GCMs,
##### Similar to Fig 1 of the paper
#########################################################
```

```
zlim<-range(B)
surf.1 <- mba.surf(cbind(coords,data=B[,1]), no.X=100, no.Y=100)$xyz.est
surf.2 <- mba.surf(cbind(coords,data=B[,2]), no.X=100, no.Y=100)$xyz.est
surf.3 <- mba.surf(cbind(coords,data=B[,3]), no.X=100, no.Y=100)$xyz.est
surf.4 <- mba.surf(cbind(coords,data=B[,4]), no.X=100, no.Y=100)$xyz.est
surf.5 <- mba.surf(cbind(coords,data=B[,5]), no.X=100, no.Y=100)$xyz.est
surf.6 <- mba.surf(cbind(coords,data=B[,6]), no.X=100, no.Y=100)$xyz.est


par(mfrow=c(2,3))
image.plot(surf.1, main=B.names[1])
image.plot(surf.2, main=B.names[2])
image.plot(surf.3, main=B.names[3])
image.plot(surf.4, main=B.names[4])
image.plot(surf.5, main=B.names[5])
image.plot(surf.6, main=B.names[6])


########################################################################
## Define Gaussian weighting kernels to model the overall bias as in the
##paper.For ease of implementation, we only use 9 Gaussian kerenels here
#############################################################################
rx<-range(coords[,1]) # range of x-coords
ry<-range(coords[,2]) # range of y-coords
ngx<-3
ngy<-3
borderx<-diff(rx)/(2*ngx)
bordery<-diff(ry)/(2*ngy)
xg<-seq(rx[1]+borderx,rx[2]-borderx,length.out = ngx)
yg<-seq(ry[1]+bordery,ry[2]-bordery,length.out = ngy)
centers<-as.matrix(expand.grid(xg,yg)) # centers of the kernels
scale<-3*borderx
Kern1<-function(center,coords,cov, inverted=FALSE) {
  d<-mahalanobis(x=coords,center=center,cov=cov,inverted = inverted)
  d<-exp(-0.5*d)
  return(d)
}

W<-apply(centers, 1, FUN = Kern1,coords=coords,cov=scale*diag(c(1,2)))
dim(W)
p<-ncol(W) # number of kernels
I <- ncol(B) # number of model outputs

########################################################################
## Define Gaussian weighting kernels to model model-specific bias as in the
##paper.For ease of implementation, we only use 12 Gaussian kerenels here
#############################################################################

rx<-range(coords[,1]) # range of x-coords
ry<-range(coords[,2]) # range of y-coords
ngx<-4
ngy<-3
borderx<-diff(rx)/(2*ngx)
bordery<-diff(ry)/(2*ngy)
xg<-seq(rx[1]+borderx,rx[2]-borderx,length.out = ngx)
yg<-seq(ry[1]+bordery,ry[2]-bordery,length.out = ngy)
centers<-as.matrix(expand.grid(xg,yg)) # centers of the kernels
scale<-2*borderx
Kern2<-function(center,coords,cov, inverted=FALSE) {
  d<-mahalanobis(x=coords,center=center,cov=cov,inverted = inverted)
  d<-exp(-0.5*d)
  return(d)
}
W2<-apply(centers, 1, FUN = Kern1,coords=coords,cov=scale*diag(c(1,2)))
dim(W2)
p2<-ncol(W2) # number of kernels
I <- ncol(B) # number of model outputs



#####################################################################
## note in the paper, W2 is denoted by w* and p2 is denoted by p*
######################################################################


##############################################
```

```
## prepare the data and store as list for Bugs
################################################

B<-as.matrix(B) ## the bias data
colnames(B)<-NULL
N_obs=nrow(B) ##number of observations
p=dim(W)[2] ## dimension of the first set of kernels
p2=dim(W2)[2] ## dimension of the secon set of kernels
I=6  ## number of climate models
df=p+1 ## degree of freedom for Wishart prior
IDE<-diag(p) ## scale matrix for wishart prior
prior.scale=30 ## hyperprior for the variance tau

Data<-list("B","W","W2","N_obs","p","p2","I","prior.scale","IDE","df")



###################################################
## Define intial values for all random parameters
###################################################

inits<-function(){list(eps=matrix(0.01,nrow=I,ncol=p2),
        tau.b=0.01,
        xi=0.01,
        tau.eps=rep(0.01,I),
        beta.raw=rep(0.01,p),
        mu.hat.raw=rep(0.01,p),
        omega=rep(0.8,p),
        Tau.beta.raw =diag(p))}


##########################################################################
## Define the model stored in the working directory as 'Model-name.txt'
## the model is written in BUG script
###########################################################################

program.file.name="Model-name.txt"

##########################################################################
## Set the parameters to be monitored after MCMC
###########################################################################

parameters=list("alpha", "std.tau")

##########################################################################
## Set number of simulation, number of thining and number of burnin
###########################################################################

n.thin=1
n.iter=4000
n.burnin=2000


##################################################################################################
## Call apriori installed BUG to perform the MCMC.Here we use OpenBUGS via the package R2OpenBUGS
## wait untile the MCMC converge
#####################################################


fitmodel<- bugs(Data, inits, parameters, model.file =program.file.name,
         n.chains = 1, n.iter = n.iter, n.burnin = n.burnin,
         n.thin = n.thin,debug = TRUE, DIC = FALSE, digits = 3, codaPkg = FALSE, useWINE=TRUE,
         working.directory= "/Users/maeregu/Dropbox/supplemental",
         WINE="/opt/local/bin/wine",
         OpenBUGS.pgm= "/Users/maeregu/.wine/drive_c/Program Files/OpenBUGS/OpenBUGS323/OpenBUGS.exe")


###################################################################################
## After MCMC convergence, we summarize posterior of parameter of interest
####################################################################################

alpha<-fitmodel$sims.list$alpha
class(alpha)
dim(alpha)
alpha.mean<-as.matrix(colMeans(alpha))
dim(alpha.mean)
```

```
##################################################
## Recover the overall common bias. Plot
## a surface similar to Fig 4(a) in the paper
###################################################

overall.bias<-as.vector(W%*%alpha.mean)
surf.mu <- mba.surf(cbind(coords,data=overall.bias), no.X=100, no.Y=100)$xyz.est
image.plot(surf.mu, main="overall common bias")
points(centers,pch=20)

#######################################################
## Summarize the individual departure from the
## overall common bias. Surfaces similar to
## Fig 5 in the paper
####################################################

eta1<-Y[,1]-overall.bias
eta2<-Y[,2]-overall.bias
eta3<-Y[,3]-overall.bias
eta4<-Y[,4]-overall.bias
eta5<-Y[,5]-overall.bias
eta6<-Y[,6]-overall.bias
surf.eta1 <- mba.surf(cbind(coords,data=eta1), no.X=100, no.Y=100)$xyz.est
surf.eta2 <- mba.surf(cbind(coords,data=eta2), no.X=100, no.Y=100)$xyz.est
surf.eta3 <- mba.surf(cbind(coords,data=eta3), no.X=100, no.Y=100)$xyz.est
surf.eta4 <- mba.surf(cbind(coords,data=eta4), no.X=100, no.Y=100)$xyz.est
surf.eta5 <- mba.surf(cbind(coords,data=eta5), no.X=100, no.Y=100)$xyz.est
surf.eta6 <- mba.surf(cbind(coords,data=eta6), no.X=100, no.Y=100)$xyz.est
par(mfrow=c(2,3))
image.plot(surf.eta1, main="contribution of CCSM4")
image.plot(surf.eta2, main="contribution of BCC")
image.plot(surf.eta3, main="contribution of IPSL")
image.plot(surf.eta4, main="contribution of MPI")
image.plot(surf.eta5, main="contribution of GISS")
image.plot(surf.eta6, main="contribution of MIROC")


############################################
###        END
############################################
```

Model-name.txt

```
model
{

 for (s in 1:N_obs)
  {

  for (m in 1:I)
   {

   B[s,m]~dnorm(M[s,m],tau.b[m])

   M[s,m]<-mu[s]+ eta[s,m]

  eta[s,m]<-W2[s,1]*nu[m,1]+W2[s,2]*nu[m,2]+W2[s,3]*nu[m,3]+W2[s,4]*nu[m,4] + W2[s,5]*nu[m,5]+
          W2[s,6]*nu[m,6]+W2[s,7]*nu[m,7]+W2[s,8]*nu[m,8]+W2[s,9]*nu[m,9] + W2[s,10]*nu[m,10]+
          W2[s,11]*nu[m,11]+W2[s,12]*nu[m,12]

      }

   mu[s]<-W[s,1]*alpha[1]+W[s,2]*alpha[2]+W[s,3]*alpha[3]+W[s,4]*alpha[4] +
        W[s,5]*alpha[5]+ W[s,6]*alpha[6]+W[s,7]*alpha[7]+W[s,8]*alpha[8]+W[s,9]*alpha[9]
      }

       for (d in 1:p2)
      {
      for (m in 1:I)
       {
```

```
        nu[m,d]<-xi*eps[m,d]
        eps[m,d]~dnorm(0,tau.eps[m])
       }
       }

    xi~dnorm(0,tau.xi)
    tau.xi<-pow(prior.scale,-2)

     for (m in 1:I)
      {
     # the half caucy prior
      tau.eps[m]~dgamma(0.5,0.5)          ## chi-squared
      std.tau[m]<-abs(xi)/sqrt(tau.eps[m]) ## inference on std which is normal r.v divide by chi-squared r.v
      }


        for (b in 1:p)
         {
           alpha[b]<-omega[b]*beta.raw[b]
         }
        beta.raw[1:p]~dmnorm(mu.hat.raw[],Tau.beta.raw[,])

          for(k in 1:p){
            mu.hat[k]<-omega[k]*mu.hat.raw[k]
            mu.hat.raw[k]~dnorm(0,0.0001)
            omega[k]~dunif(0,100)
             }

         Tau.beta.raw[1:p,1:p] ~ dwish(IDE[,], df)
         Sigma.beta.raw[1:p,1:p] <- inverse(Tau.beta.raw[,])

         for (k in 1:p){
          for (l in 1:p){
           rho.beta[k,l]<-Sigma.beta.raw[k,l]/sqrt(Sigma.beta.raw[k,k]*Sigma.beta.raw[l,l])
           }
           sigma.beta[k] <- abs(omega[k])*sqrt(Sigma.beta.raw[k,k])
            }

for (m in 1:I){
tau.b[m]~dunif(0,100)}

}#End model
```

Data-bias.txt

```
"V1","V2","V3","V4","V5","V6"
1.82294648996175,4.35935211253508,-1.05349011673421,-0.149924424102494,0.271088056445915,2.488589568562
1.21838063791373,4.27639037103832,-1.59353999492305,-0.400131333216301,-0.088541516051901,2.23473814702036
1.27998461948831,4.74124444824598,-1.42973395007578,-0.504735403210663,0.0181302649414725,2.66591737484245
1.42981153086706,4.96924719724484,-1.30623251439391,-0.399979580494925,0.245369991421047,2.88224631880957
1.14581241055413,4.41595637215013,-1.74122443392526,-0.604584183410104,0.0446097686323981,2.34334374448843
1.35767890819642,3.98180051202656,-1.54926662110427,-0.509345688763561,0.347073341382607,1.96570014972171
1.42615757756832,3.02359103599303,-1.67518175204624,-0.340552943975359,0.521115681643497,1.0743537777095
1.18802679929447,1.86147264145631,-2.28178176986238,-0.552329911966979,0.431166847218499,-0.148388691220774
1.26819363097246,3.20219394053592,-2.30491880892546,-0.363412001177084,0.68172976340588,0.926419710763863
1.22773230037509,4.31251432051743,-2.43074424495408,-0.11339998633018,0.793610252529735,1.91087473507849
1.07590379162593,4.84584058507591,-2.69942393429356,-0.393379890536806,0.739154114286372,2.40476804888835
1.04950794103934,5.12611039974146,-2.6975396661985,-0.477256076177602,0.73736703220661,2.64300638720718
1.15302953094538,5.21901082000926,-2.7724999250922,-0.209148240817452,0.79992943990419,2.66630171035652
1.40332730459143,5.19040899157003,-2.85043970827168,-0.0387787633611119,0.976899642819575,2.53428994990821
1.36917681661048,4.55518403828734,-2.8654218662992,0.143366829533644,0.812814407129064,1.77571281162844
1.31679024013332,3.41982155881743,-3.08990303901372,0.434869851182182,0.57086789162766,0.560075002415836
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Lat.txt

-35
-32.5
-30
-27.5
-25
-22.5
-20
-17.5
-15
-12.5
-10
-7.5
-5
-2.5
0
2.5
5
7.5
10
12.5
15

Lon.txt

-40
-37.5
-35
-32.5
-30
-27.5
-25
-22.5
-20
-17.5
-15
-12.5
-10
-7.5
-5
-2.5
0

2.5
5
7.5
10
12.5
15
17.5
20

Appendix 3

R-code to perform Bayesian hierarchical spatio-temporal assessment of the bias in a multi-model mean ensemble described in section 3 of this deliverable.

The file 'R-code.txt' is the main R code to run the model. The file provides indications about the necessary installation of software packages. The code in ‘R-code.txt’ should be copy/pasted to the R shell to run the codes.

The code refers to a data file called “bias_1948-2005_ym.nc”, which is not provided. Any netcdf file containing 3-dimensional data with dimensions longitude, latitude, time can be used straightforwardly. The code can be easily modified to use other data sources.

R-code.txt:

```
#####################################
## Install appropriate Packages
####################################

library(geoR)
library(fields)
library(maps)
library(grid)
library(gridExtra)
library("MBA")
library(plot3D)
library("spBayes")
require(devtools)
library(ncdf4)



#####################################################
### read the data and prepare the data for the
## analysis
######################################################
sptembensbiasnc<-nc_open("bias_1948-2005_ym.nc")
sptembensbias <- ncvar_get(sptembensbiasnc,"tas")
sptembensbias[sptembensbias=="1e+30"] <- NA
##### lon lat dimensio
lat<-sptembensbiasnc$dim$lat$vals
lat<- round(rev(lat),digits=0)
lon<-round(sptembensbiasnc$dim$lon$vals,digits=0)
# Set lon [-180 177.5]
k=which(lon==180)
px=c(k:length(lon),1:(k-1))
lon=lon[px]
lon[1:(which(lon==0.0))-1]=lon[1:(which(lon==0.0)-1)]-360
######  on the correct lat lon scale
sptembensbias<-sptembensbias[,ncol(sptembensbias):1,]
sptembensbias=sptembensbias[px,,]
class(sptembensbias)
dim(sptembensbias)

######################################################
## limit the spatial area to the tropical Atlantic
######################################################

lontrop<-lon[match(-50,lon):match(30,lon)]
lattrop<-lat[match(-40,lat):match(40,lat)]
sptembiastrop=sptembensbias[match(-50,lon):match(30,lon),match(-40,lat):match(40,lat),]


######################################################
## create the grid points
######################################################
```

```
coords<-as.matrix(expand.grid(lontrop,lattrop))
class(coords)
dim(coords)

############################################
##aggregate over space to identify the overall
## time series trend
#############################################

sptembiasvect<-NULL
for (j in 1:58){
  sptembiasvect[[j]]<-as.vector(sptembiastrop[,,j])
}
class(sptembiasvect)

sptembiasdata<-t(as.data.frame(sptembiasvect))
class(sptembiasdata)
dim(sptembiasdata)

tempbias<-apply(sptembiasdata, 1, mean)
class(tempbias)
length(tempbias)

tseries <- ts(tempbias, start=1948, end=2005, frequency=1)
plot(tseries,lwd=2, ylab= "climate model bias", xlab="time")
abline(h=0,col=6)


#################################################
##aggregate over time to identify the empirical
## spatial patterns
#################################################

spbias<-apply(sptembiasdata, 2, mean)
class(spbias)
length(spbias)

range(spbias)
spat.surf<- mba.surf(cbind(coords,data=spbias), no.X=60, no.Y=60)$xyz.est
image.plot(spat.surf,xlab="longitude",ylab="latitude")
map(add=T)



########################################################
##Define a vector of Gaussian kernels with length 16
########################################################

rx<-range(coords[,1]) # range of x-coords
ry<-range(coords[,2]) # range of y-coords
ngx<-4 #
ngy<-4 # ngx*ngy number of kernels
borderx<-diff(rx)/(2*ngx)
bordery<-diff(ry)/(2*ngy)
xg<-seq(rx[1]+borderx,rx[2]-borderx,length.out = ngx)
yg<-seq(ry[1]+bordery,ry[2]-bordery,length.out = ngy)
centers<-as.matrix(expand.grid(xg,yg)) # centers of the kernels
scale<-3*borderx


kw<-function(center,coords,cov, inverted=FALSE) {
  d<-mahalanobis(x=coords,center=center,cov=cov,inverted = inverted)
  d<-exp(-0.5*d)
  return(d)
}

K<-apply(centers, 1, FUN = kw,coords=coords,cov=scale*diag(c(1,2)))


########################################################
##The kernel matrix in the spatio-temporal form
########################################################

kernmat<-K[,rep(1:ncol(K), each=58)]
class(kernmat)
dim(kernmat)
kernmat<-as.data.frame(kernmat)
```

```
colnames(kernmat)
colnames(kernmat)<-NA
colnames(kernmat)[1:58]<- paste("K1.", 1:58,sep="")
colnames(kernmat)[59:116]<- paste("K2.", 1:58,sep="")
colnames(kernmat)[117:174]<-paste("K3.", 1:58,sep="")
colnames(kernmat)[175:232]<-paste("K4.", 1:58,sep="")
colnames(kernmat)[233:290]<-paste("K5.", 1:58,sep="")
colnames(kernmat)[291:348]<-paste("K6.", 1:58,sep="")
colnames(kernmat)[349:406]<-paste("K7.", 1:58,sep="")
colnames(kernmat)[407:464]<-paste("K8.", 1:58,sep="")
colnames(kernmat)[465:522]<-paste("K9.", 1:58,sep="")
colnames(kernmat)[523:580]<-paste("K10.", 1:58,sep="")
colnames(kernmat)[581:638]<-paste("K11.", 1:58,sep="")
colnames(kernmat)[639:696]<-paste("K12.", 1:58,sep="")
colnames(kernmat)[697:754]<-paste("K13.", 1:58,sep="")
colnames(kernmat)[755:812]<-paste("K14.", 1:58,sep="")
colnames(kernmat)[813:870]<-paste("K15.", 1:58,sep="")
colnames(kernmat)[871:928]<-paste("K16.", 1:58,sep="")
dim(kernmat)
names(kernmat)


#######################################################
### form a spatiotemporal data frame for the biases
########################################################

sptembiasvect<-NULL
for (j in 1:58){
  sptembiasvect[[j]]<-as.vector(sptembiastrop[,,j])
}
class(sptembiasvect)

sptemdata<-as.data.frame(sptembiasvect)
class(sptemdata)
dim(sptemdata)

names(sptemdata)
colnames(sptemdata)<-NULL
colnames(sptemdata) <- paste("B.", 1:58,sep="")



geo<-data.frame(lon=coords[,1],lat=coords[,2])
class(geo)
dim(geo)


#####################################
### combine the datasets and export
######################################

ensptempData16k<-cbind(geo,sptemdata,kernmat)
class(ensptempData16k)
dim(ensptempData16k)
names(ensptempData16k)


#####################################################
#####################################################
### Now specify the spatio-temporal model
#####################################################
#####################################################

Nt=58## number of years
n<-nrow(ensptempData16k) ##number of observation per months
maxd <- max(iDist(coords))
##set starting and priors
p<- 17 #number of kernel coefficients parameters in each month

starting <-list("beta"=rep(0,Nt*p), "phi"=rep(3/(0.5*maxd), Nt),
             "sigma.sq"=rep(2,Nt), "tau.sq"=rep(1, Nt),
             "sigma.eta"=diag(rep(0.01, p)))

tuning <- list("phi"=rep(5, Nt))

priors <- list("beta.0.Norm"=list(rep(0,p), diag(1000,p)),
            "phi.Unif"=list(rep(3/(0.9*maxd), Nt), rep(3/(0.05*maxd), Nt)),
```

```
        "sigma.sq.IG"=list(rep(2,Nt), rep(10,Nt)),
        "tau.sq.IG"=list(rep(2,Nt), rep(5,Nt)),
        "sigma.eta.IW"=list(2, diag(0.001,p)))

##make symbolic model formula statement for each month
models<-lapply(paste("B.",1:Nt, "~K1.",1:Nt,"+K2.",1:Nt,"+K3.",1:Nt,"+K4.",1:Nt,"+K5.",1:Nt,"+K6.",1:Nt,"+K7.",1:Nt,"+K8.",
          1:Nt,"+K9.",1:Nt,"+K10.",1:Nt,"+K11.",1:Nt,"+K12.",1:Nt,"+K13.",1:Nt,"+K14.",1:Nt,"+K15.",1:Nt,"+K16.",
          1:Nt, sep=""),as.formula)
nsamples<-2000


sptemodel<-spDynLM(models,data=ensptempData16k,coords=coords,starting=starting,
         tuning=tuning, priors=priors,cov.model="exponential",
         n.samples=nsamples)



####################################################
####################################################
###Summarize the posterior information
####################################################
####################################################


burnin <- floor(0.30*nsamples)
n=1089

betasim<-sptemodel$p.beta.samples
class(betasim)
dim(betasim)


quant <- function(x){quantile(x, prob=c(0.5, 0.025, 0.975))}
beta <- apply(sptemodel$p.beta.samples[burnin:nsamples,], 2, quant)
class(beta)
dim(beta)
colnames(beta)
beta.0 <- beta[,grep("Intercept", colnames(beta))]
beta.1 <- beta[,grep("K", colnames(beta))]


class(beta.1)
dim(beta.1)
colnames(beta.1)


arr<-as.vector(NA)
d=1
a=16
for(m in 2: 58){
  arr[d]=1
  arr[m]=arr[m-1]+a
}

class(arr)
length(arr)

betat<-list(NA)
betatmat<-list(NA)
Ms<-list(NA)
Msmat<-list(NA)
J=15


dim(K) ## use the kernel matrix to recover the process M

for (v in 1:58){
  betat[[v]]<-beta.1[,arr[v]:(arr[v]+J)] ## collect a list of time trend matrix in the list class
  betatmat[[v]]<-matrix(c(betat[[v]][d,])) ## a vector of kernel coefficients size 16 for each time
  Ms[[v]]<-K%*%betatmat[[v]] ### recover the process by multpling with the fixed kernel matrix
  Msmat[[v]]<-matrix(Ms[[v]], nrow=length(lontrop),ncol=length(lattrop),byrow=F) ## create lon by lat matrix of the recovered process for posterior plot
}



Ms.surf1<- mba.surf(cbind(coords,data=as.vector(Ms[[3]])), no.X=100, no.Y=100)$xyz.est
Ms.surf2<- mba.surf(cbind(coords,data=as.vector(Ms[[13]])), no.X=100, no.Y=100)$xyz.est
```

```
Ms.surf3<- mba.surf(cbind(coords,data=as.vector(Ms[[23]])), no.X=100, no.Y=100)$xyz.est
Ms.surf4<- mba.surf(cbind(coords,data=as.vector(Ms[[33]])), no.X=100, no.Y=100)$xyz.est
Ms.surf5<- mba.surf(cbind(coords,data=as.vector(Ms[[43]])), no.X=100, no.Y=100)$xyz.est
Ms.surf6<- mba.surf(cbind(coords,data=as.vector(Ms[[53]])), no.X=100, no.Y=100)$xyz.est


par(mforw=c(2,3))
image2D(Ms.surf1)
image2D(Ms.surf2)
image2D(Ms.surf3)
image2D(Ms.surf4)
image2D(Ms.surf5)
image2D(Ms.surf6)
```